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On the Nonequilibrium Statistical Mechanics of a Binary Mixture. II. The Transport Coefficients

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This paper is devoted to the study of the hydrodynamic stage of a two-component dense fluid. Starting from the BBGKY hierarchy obtained earlier, we first derive the expressions for the generalized fluxes. We proceed to set up the generalized kinetic equations, using Bogoliubov's functional assumption. Then we solve these equations by means of a Chapman–Enskog method. The generalized expressions for the transport coefficients are thus obtained. All our results are independent of the existence of density expansions of the relevant quantities.

KEY WORDS: Transport coefficients for a binary mixture; convergent kinetic theory; fluxes and conjugated forces for a binary mixture; Soret coefficient; Dufour coefficient; shear and bulk viscosities for a binary mixture.

1. INTRODUCTION

This paper is devoted to the study of the linear transport coefficients for a binary mixture of dense gases. The purpose is to obtain general expressions for these quantities without reference to a density expansion. In this sense, it is a generalization of the work done for a one-component dense gas by

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by García–Colín, Green, and Chaos (GCGC).[1] Indeed, we start out from the set of equations for the single-particle distribution functions of each species which have been previously derived.[3] These equations together with the Bogoliubov assumption that the distribution functions of more than one particle are time-independent functionals of the single-particle distribution functions, yield the two coupled kinetic equations of our system. After linearization of the kinetic equations in the gradients of the system, we set to solve them following the standard Chapman–Enskog method. In this way, we obtain the general expressions for the transport coefficients in terms of the intermolecular potentials.

In Section 2, we outline the derivation of the hydrodynamic equations for a binary mixture starting from the BBGKY hierarchy. In Section 3, the generalized kinetic equations are linearized in the gradients, a result which is used together with the hydrodynamic equations in Section 4 to solve for the perturbation functions following the Chapman–Enskog method. Due to the fact that the solutions obtained in Section 4 are given in terms of the gradients of macroscopic variables which are not the conjugate ones of the hydrodynamic fluxes, a transformation to the appropriate gradients must be made. This point is discussed in Section 5. Finally, in Section 6, we obtain the explicit formulas for the transport coefficients. These formulas are given in terms of functions which still obey linear inhomogeneous integral equations that can be explicitly solved when a specific model for the intermolecular potential is chosen. The structure of these coefficients within the context of the new ideas proposed by two of us[3] which yield divergenceless terms in their density expansions will be given in a forthcoming publication.

2. HYDRODYNAMIC EQUATIONS FOR A BINARY MIXTURE

In this section, we will sketch the way in which the hydrodynamic equations for a binary mixture are obtained.

The first and second equations of the BBGKY hierarchy for the binary mixture were obtained in I. They can be written as

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \cdot \frac{\partial}{\partial \mathbf{q}_{\gamma 1}}\right) f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1})$$

$$= \sum_{\alpha=1}^{2} \int dx_{\alpha,\xi(\alpha,\gamma)} \theta_{\gamma\alpha}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}) f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}; t),$$

$$\gamma = 1, 2 \quad (1)$$

[3] Hereafter referred to as I. We use the same notation as in I.

and

$$\left(\frac{\partial}{\partial t} + \mathscr{H}_{\{s_a\}\{s_b\}}\right) f_{\{s_a\}\{s_b\}}$$
$$= \sum_{\beta=1}^{2} \sum_{\alpha=1}^{2} \sum_{i=1}^{s_a} \int dx_{\beta, s_\beta+1} \theta_{\alpha\beta}(x_{\alpha i}, x_{\beta, s_\beta+1}) f_{\{s_a-\beta+2\}\{s_b+\beta-1\}}$$
$$s_a, s_b = 0, 1, 2, \qquad s_a + s_b = 2 \tag{2}$$

Here we have used the distribution function $f_{\{s_a\}\{s_b\}}$ defined by

$$f_{\{s_a\}\{s_b\}} = n_a^{s_a} n_b^{s_b} F_{\{s_a\}\{s_b\}} \tag{3}$$

The rest of the symbols were defined in I.

From these equations, we can obtain the hydrodynamic equations for the binary mixture following the standard technique first outlined by Choh and Uhlenbeck. In fact, multiplying Eq. (1) by 1, $\mathbf{p}_{\gamma 1}$, and $(\mathbf{p}_{\gamma 1}^2/2m_\gamma)$ and integrating the resulting expressions with respect to $\mathbf{p}_{\gamma 1}$, we obtain the continuity equations, the equation of motion, and the equation for the conservation of kinetic energy, respectively, given by

$$\partial n_\alpha/\partial t = -\operatorname{div}(n_\alpha \mathbf{u}) - \operatorname{div}(\mathbf{J}_\alpha/m_\alpha) \tag{4a}$$

$$\partial \rho/\partial t = -\operatorname{div}(\rho \mathbf{u}) \tag{4b}$$

$$\rho\, D\mathbf{u}/Dt = -\operatorname{div} \mathsf{P}$$

$$\frac{D}{Dt}\left(\frac{3}{2}\theta\right) + \operatorname{div} \mathbf{j}^k + \mathsf{P} : \mathsf{D} - \operatorname{div}(\mathbf{u} \cdot \mathsf{P}^\varphi) \tag{5}$$

$$= -\sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \frac{\partial \varphi_{\alpha\gamma}}{\partial \mathbf{q}_{\gamma 1}} \cdot \frac{\mathbf{p}_{\gamma 1}}{m_\gamma}$$
$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}; t) \tag{6}$$

In these equations, we have used the expressions

$$n_\gamma = \int d\mathbf{p}_{\gamma 1} f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}), \qquad \gamma = 1, 2 \tag{7}$$

$$\mathbf{J}_\gamma = \int d\mathbf{p}_{\gamma 1} (\mathbf{p}_{\gamma 1}/m_\gamma) f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}), \qquad \gamma = 1, 2 \tag{8}$$

$$\mathbf{u} = (1/\rho) \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma 1}\, \mathbf{p}_{\gamma 1} f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) \tag{9}$$

$$\tfrac{3}{2} n\theta = \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma 1} (p_{\gamma 1}^2/2m_\gamma) f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) \tag{10}$$

which are the definitions of the particle density and mass flux of each species, the velocity of the center of mass of the system, and the temperature, respectively.

As usual, the mass density is given by $\rho_\gamma = m_\gamma n_\gamma$ ($\gamma = 1, 2$), the total mass density by $\rho = \sum_{\gamma=1}^{2} \rho_\gamma$, and the total number density by $n = \sum_{\gamma=1}^{2} n_\gamma$. Moreover, we have introduced $\boldsymbol{p\!\!/}_{\gamma i} = \mathbf{p}_{\gamma i} - m_\gamma \mathbf{u}$, the operator $D/Dt \equiv (\partial/\partial t) + \mathbf{u} \cdot \text{grad}$, and the deformation tensor D whose components are $D_{ij} = (\frac{1}{2})[(\partial u_i/\partial q_j) + (\partial u_j/\partial q_i)]$. We have also written

$$\xi(\alpha, \gamma) = (\alpha - 2)(2\gamma - 3) + \gamma \qquad (\alpha, \gamma = 1, 2).$$

The stress tensor is given by

$$\mathsf{P} = \mathsf{P}^k + \mathsf{P}^\varphi \tag{11}$$

with

$$\mathsf{P}^k = \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma 1} \frac{\boldsymbol{p\!\!/}_{\gamma 1}\boldsymbol{p\!\!/}_{\gamma 1}}{m_\gamma} f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) \tag{12}$$

$$\mathsf{P}^\varphi = -\sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \frac{1}{2} \int d\mathbf{r}_{\gamma\alpha} \frac{\mathbf{r}_{\gamma\alpha}\mathbf{r}_{\gamma\alpha}}{r_{\gamma\alpha}} \varphi'_{\gamma\alpha}(r_{\gamma\alpha}) \int d\mathbf{p}_{\gamma 1} \int d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)} \int_0^1 d\mu$$
$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(\mathbf{q}_{\gamma 1} - \mu\mathbf{r}_{\gamma\alpha}, \mathbf{q}_{\gamma 1} + (1-\mu)\,\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}; t) \tag{13}$$

Here, $\mathbf{r}_{\gamma\alpha} \equiv \mathbf{q}_{\gamma 1} - \mathbf{q}_{\alpha,\xi(\alpha,\gamma)}$, and $\varphi'_{\gamma\alpha}$ denotes the derivative of $\varphi_{\gamma\alpha}$ with respect to its argument.

The kinetic contribution to the heat flux $\mathbf{j}^k$ is expressed as

$$\mathbf{j}^k = \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma 1} \frac{\boldsymbol{p\!\!/}_{\gamma 1}}{m_\gamma} \frac{\boldsymbol{p\!\!/}_{\gamma 1}^2}{2m_\gamma} f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}\} \tag{14}$$

Multiplying Eq. (2) by $\frac{1}{2}\varphi_{\gamma\alpha}$, integrating over $\mathbf{p}_{\gamma 1}$ and $x_{\alpha,\xi(\alpha,\gamma)}$, and summing over γ and α, we find that

$$\frac{\partial}{\partial t}(n\epsilon^\varphi) + \frac{1}{2} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)}\, \varphi_{\gamma\alpha}(r_{\gamma\alpha})$$
$$\times \left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \cdot \frac{\partial}{\partial \mathbf{q}_{\gamma 1}} + \frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha} \cdot \frac{\partial}{\partial \mathbf{q}_{\alpha,\xi(\alpha,\gamma)}}\right)$$
$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}; t) = 0 \tag{15}$$

Here,

$$n\epsilon^\varphi = \frac{1}{2} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)}\, \varphi_{\gamma\alpha}(r_{\gamma\alpha})$$
$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}; t) \tag{16}$$

Adding Eqs. (6) and (15), we find the total energy equation

$$n(D\epsilon/Dt) + \operatorname{div} \mathbf{j} = -\mathsf{P} : \mathsf{D} \tag{17}$$

where the total heat flux $\mathbf{j}$ is

$$\mathbf{j} = \mathbf{j}^k + \mathbf{j}^{\varphi_1} + \mathbf{j}^{\varphi_2} \tag{18}$$

with

$$\mathbf{j}^{\varphi_1} = \frac{1}{4} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \frac{\varphi'_{\gamma\alpha}(r_{\gamma\alpha})}{r_{\gamma\alpha}} \mathbf{r}_{\gamma\alpha}\mathbf{r}_{\gamma\alpha} \cdot \left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma} + \frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha}\right)$$
$$\times \int_0^1 d\mu\, f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(\mathbf{q}_{\gamma 1} - \mu \mathbf{r}_{\gamma\alpha}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}(1-\mu), \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}; t) \tag{19}$$

$$\mathbf{j}^{\varphi_2} = \frac{1}{2} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \varphi_{\gamma\alpha}(r_{\gamma\alpha})$$
$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}; t) \tag{20}$$

and $\mathbf{j}^k$ given by Eq. (14).

The total energy density ϵ is

$$\epsilon = \tfrac{3}{2}\theta + \epsilon^{\varphi} \tag{21}$$

3. KINETIC EQUATIONS

In order to obtain the kinetic equations for a binary mixture of dense gases, we proceed in a similar way as is done for a one-component gas.[1] In fact, we assume that in the evolution of the mixture toward-its equilibrium state, there exists a stage, the so-called "kinetic stage," in which the distribution functions of two or more particles are time-independent functionals of the two single-particle distribution functions. That is,

$$f_{\{s_a\}\{s_b\}}(\cdots; t) \to f_{\{s_a\}\{s_b\}}(\cdots \mid f_{\{1\}\{0\}}, f_{\{0\}\{1\}}), \qquad s_a, s_b \geqslant 0, \quad s_a + s_b \geqslant 2 \tag{22}$$

We would like to stress that the explicit forms of these functionals have been obtained in I as power series in the density. However in this paper, we will not use these explicit forms, because we want to obtain general expressions for the kinetic equations and linear transport coefficients without any reference to a density expansion.

Substituting these relations into Eq. (1) one finds that

$$\begin{aligned}\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}_{\gamma 1}}{m_\gamma}\cdot\frac{\partial}{\partial \mathbf{q}_{\gamma 1}}\right) f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) \\ = \sum_{\alpha=1}^{2} \int dx_{\alpha,\xi(\alpha,\gamma)}\, \theta_{\gamma\alpha}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)}) f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(x_{\gamma 1}, \\ x_{\alpha,\xi(\alpha,\gamma)} \mid f_{\{1\}\{0\}}, f_{\{0\}\{1\}}) \\ \equiv \sum_{\alpha=1}^{2} \Phi_{\gamma\alpha}(x_{\gamma 1} \mid f_{\{1\}\{0\}}, f_{\{0\}\{1\}}), \qquad \gamma = 1, 2\end{aligned} \tag{23}$$

It should be noticed that the two equations (23) form a closed set of coupled equations for $f_{\{1\}\{0\}}$ and $f_{\{0\}\{1\}}$, and they constitute the kinetic equations for our system.

Due to the fact that it is our purpose to obtain linear transport coefficients, we proceed to localize the kinetic equations. We obtain the following results:

$$\begin{aligned}\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}_{\gamma 1}}{m_\gamma}\cdot\frac{\partial}{\partial \mathbf{q}_{\gamma 1}}\right) f_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) \\ = \sum_{\alpha=1}^{2} \Phi_{\gamma\alpha}(x_{\gamma 1} \mid f_{\{1\}\{0\}}(\mathbf{q}_{\gamma 1}), f_{\{0\}\{1\}}(\mathbf{q}_{\gamma 1})) \\ + \sum_{\alpha=1}^{2}\sum_{\nu=1}^{2} \int dx'\, \Phi'_{\gamma\alpha,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f_{\{1\}\{0\}}(\mathbf{q}_{\gamma 1}), f_{\{0\}\{1\}}(\mathbf{q}_{\gamma 1})) \\ \times (\mathbf{q}' - \mathbf{q}_{\gamma 1}) \cdot \left(\frac{\partial f_{\{2-\nu\}\{\nu-1\}}(x')}{\partial \mathbf{q}^1}\right)_{\mathbf{q}'=\mathbf{q}_{\gamma 1}}, \qquad \gamma = 1, 2\end{aligned} \tag{24}$$

In these expressions

$$\Phi'_{\gamma\alpha,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f_{\{1\}\{0\}}(\mathbf{q}_{\gamma 1}), f_{\{0\}\{1\}}(\mathbf{q}_{\gamma 1}))$$

is the functional derivative of $\Phi_{\gamma\alpha}(x_{\gamma 1} \mid f_{\{1\}\{0\}}, f_{\{0\}\{1\}})$ with respect to the single distribution function $f_{\{2-\nu\}\{\nu-1\}}$ taken at the phase point x'.

In the next section, we will solve Eqs. (24) using the Chapman–Enskog method.

4. THE CHAPMAN–ENSKOG SOLUTIONS OF THE KINETIC EQUATIONS

We will use the Chapman–Enskog method in order to solve the kinetic equations for the mixture obtained in Section 3. For this purpose, we expand the one-body distribution function in a power series of the macroscopic gradients of the system, and keep only linear terms:

$$f_{\{2-\gamma\}\{\gamma-1\}} = f^e_{\{2-\gamma\}\{\gamma-1\}}(1 + \phi_{\{2-\gamma\}\{\gamma-1\}}), \qquad \gamma = 1, 2 \tag{25}$$

where $f^e_{\{2-\gamma\}\{\gamma-1\}}$ is independent of the gradients.

Substituting Eqs. (25) into Eqs. (23), (7)–(10), (11), (17), and (18), one finds the following results.

4.1. Zeroth Order in the Gradients

To this order, one obtains from the kinetic equations

$$\sum_{\alpha=1}^{2} \Phi_{\gamma\alpha}(x_{\gamma1} \mid f^e_{\{1\}\{0\}}(\mathbf{q}_{\gamma1}), f^e_{\{0\}\{1\}}(\mathbf{q}_{\gamma1})) = 0, \qquad \gamma = 1, 2 \tag{26}$$

As usual, the solutions to these equations are given by the Maxwellian distribution functions

$$f^e_{\{2-\gamma\}\{\gamma-1\}}(x) = n_\gamma \left(\frac{m_\gamma}{2\pi\theta}\right)^{3/2} \exp\left(-\frac{\not{p}_\gamma^{\,2}}{2m_\gamma\theta}\right), \qquad \gamma = 1, 2 \tag{27}$$

with $\not{p}_\gamma = \mathbf{p} - m_\gamma \mathbf{u}$.

Furthermore, one finds that the stress tensor is given by $\mathsf{P} = p\mathsf{I}$, with

$$p = n\theta - \tfrac{1}{6} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma1}\, d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}\, d\mathbf{r}_{\gamma\alpha}\, \varphi'_{\gamma\alpha}(r_{\gamma\alpha})$$

$$\times f_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\}}(\mathbf{q}_{\gamma1}, \mathbf{q}_{\gamma1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)} \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \tag{28}$$

where I is the unit tensor. To this approximation, all other fluxes vanish and therefore the Euler equations for the binary mixture are

$$\partial n_\gamma/\partial t = -\mathrm{div}(n_\gamma \mathbf{u}), \qquad \gamma = 1, 2 \tag{29}$$

$$\partial \mathbf{u}/\partial t = -\mathbf{u} \cdot \mathrm{grad}\, \mathbf{u} - (1/\rho)\, \mathrm{grad}\, p \tag{30}$$

$$\partial \epsilon/\partial t = -\mathbf{u} \cdot \mathrm{grad}\, \epsilon - (\epsilon + p)\, \mathrm{div}\, \mathbf{u} \tag{31}$$

Finally, one obtains that the local macroscopic densities are

$$n_\gamma = \int d\mathbf{p}_{\gamma1} f^e_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma1}), \qquad \gamma = 1, 2 \tag{32}$$

$$\mathbf{u} = (1/\rho) \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma1}\, \mathbf{p}_{\gamma1} f^e_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma1}) \tag{33}$$

$$\epsilon = \sum_{\gamma=1}^{2} \int d\mathbf{p}_{\gamma 1} \frac{\mathbf{p}_{\gamma 1}^2}{2m_\gamma} f^e_{\{2-\gamma\}\{\gamma-1\}}(x_{\gamma 1}) + \frac{1}{2} \sum_{\gamma=1}^{2} \sum_{\alpha=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \times \varphi_{\gamma\alpha}(r_{\gamma\alpha}) f^e_{\{4-\gamma-\alpha\}\{\alpha+\gamma-2\}}(x_{\gamma 1}, x_{\alpha,\xi(\alpha,\gamma)} \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \tag{34}$$

4.2. First Order in the Gradients

To first order in the gradients, one obtains the following set of integral equations for $\phi_{\{2-\gamma\}\{\gamma-1\}}$:

$$\sum_{\alpha=1}^{2} \mathbf{A}_{\gamma\alpha} \cdot \operatorname{grad} \ln n_\alpha + \mathbf{C}_\gamma \cdot \operatorname{grad} \ln \theta + D_\gamma \operatorname{div} \mathbf{u} + \mathsf{F}_\gamma : (\operatorname{grad} \mathbf{u})_s = \sum_{\nu=1}^{2} \int d\mathbf{p}' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}') \, \phi_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}'), \qquad \gamma = 1, 2 \tag{35}$$

Here, we have written

$$\mathbf{A}_{\gamma\alpha} = f^e_{\{2-\gamma\}\{\gamma-1\}} \mathbf{p}_{\gamma 1} \left[\frac{\delta_{\gamma\alpha}}{m_\gamma} + \frac{n_\alpha}{\rho\theta} \left(\frac{\partial p}{\partial n_\alpha} \right)_\theta \right] - \sum_{\nu=1}^{2} \int dx' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times (\mathbf{q}' - \mathbf{q}_{\gamma 1}) f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{p}'), \qquad \gamma, \alpha = 1, 2 \tag{36}$$

$$\mathbf{C}_\gamma = f^e_{\{2-\gamma\}\{\gamma-1\}} \frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \left[\frac{1}{\theta} \frac{\mathbf{p}_{\gamma 1}^2}{2m_\gamma} - \frac{3}{2} + \frac{m_\gamma}{\rho} \left(\frac{\partial p}{\partial \theta} \right)_{n_\nu} \right] - \sum_{\nu=1}^{2} \int dx' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times (\mathbf{q}' - \mathbf{q}_{\gamma 1}) f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{p}') \left(\frac{\mathbf{p}'^2_{a1}}{2m_a\theta} + \frac{\mathbf{p}'^2_{b1}}{2m_b\theta} - \frac{3}{2} \right), \qquad \gamma = 1, 2 \tag{37}$$

$$D_\gamma = f^e_{\{2-\gamma\}\{\gamma-1\}} \left[\frac{\mathbf{p}_{\gamma 1}^2}{3m_\gamma\theta} - \frac{3}{2} \left(\frac{1}{3\theta} \frac{\mathbf{p}_{\gamma 1}^2}{m_\gamma} - 1 \right) \frac{\partial p}{\partial \rho} \right] - \frac{1}{3\theta} \sum_{\nu=1}^{2} \int dx' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{p}')(\mathbf{q}' - \mathbf{q}_{\gamma 1}) \cdot (\mathbf{p}_{a1} + \mathbf{p}_{b1}), \qquad \gamma = 1, 2 \tag{38}$$

$$\mathsf{F}_\gamma = f^e_{\{2-\gamma\}\{\gamma-1\}} \frac{1}{m_\gamma \theta} (\mathbf{p}_{\gamma 1}\mathbf{p}_{\gamma 1})_s$$
$$- \frac{1}{\theta} \sum_{\nu=1}^{2} \int dx' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x_{\gamma 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$
$$\times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{p}')\{\tfrac{1}{2}[(\mathbf{q}' - \mathbf{q}_{\gamma 1})\, \mathbf{p}'_{\gamma 1} + \mathbf{p}'_{\gamma 1}(\mathbf{q}' - \mathbf{q}_{\gamma 1})]$$
$$- \tfrac{1}{3}\mathbf{p}'_{\gamma 1} \cdot (\mathbf{q}' - \mathbf{q}_{\gamma 1})\, \mathsf{I}\}, \qquad \gamma = 1, 2 \tag{39}$$

and

$$M_{\gamma,\{2-\nu\}\{\nu-1\}} = \sum_{\alpha=1}^{2} \Phi'_{\gamma\alpha,\{2-\nu\}\{\nu-1\}}, \qquad \gamma, \nu = 1, 2 \tag{40}$$

In Eqs. (35) and (39) the symbols (grad $\mathbf{u}$)$_s$ and $(\mathbf{pp})_s$ denote the symmetric traceless parts of the tensor (grad $\mathbf{u}$) and $(\mathbf{pp})$, respectively.

The set of linear inhomogeneous integral equations (35) for the perturbation functions $\phi_{\{2-\gamma\}\{\gamma-1\}}$ has to be solved subject to the following subsidiary conditions:

$$(1/m_\gamma) \int d\mathbf{p}\, f^e_{\{2-\gamma\}\{\gamma-1\}} \phi_{\{2-\gamma\}\{\gamma-1\}} = 0, \qquad \gamma = 1, 2 \tag{41}$$

$$\sum_{\gamma=1}^{2} (1/m_\gamma) \int d\mathbf{p}\, \mathbf{p} f^e_{\{2-\gamma\}\{\gamma-1\}} \phi_{\{2-\gamma\}\{\gamma-1\}} = 0 \tag{42}$$

and

$$\sum_{\nu=1}^{2} \int d\mathbf{p}'\, \epsilon'_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\nu 1}, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$
$$\times\, \phi_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\nu 1}, \mathbf{p}') f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\nu 1}, \mathbf{p}')$$
$$+ \sum_{\nu=1}^{2} \int dx'\, \epsilon'_{\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\nu 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$
$$\times (\mathbf{q}' - \mathbf{q}_{\nu 1}) \cdot \left(\frac{\partial f^e_{\{2-\nu\}\{\nu-1\}}(x')}{\partial \mathbf{q}'}\right)_{\mathbf{q}'=\mathbf{q}_{\nu 1}} = 0 \tag{43}$$

Here,

$$\epsilon'_{\{2-\nu\}\{\nu-1\}}\left(\mathbf{q}_{\nu 1}, \begin{pmatrix}\mathbf{p}'\\ x'\end{pmatrix} \middle|\, \cdots\right)$$

denotes the functional derivative of $\epsilon(q_{\nu 1} \mid \cdots)$ with respect to the one-particle distribution function $f_{\{2-\nu\}\{\nu-1\}}$ taken at point

$$\begin{pmatrix}\mathbf{p}'\\ x'\end{pmatrix}$$

As it has alrealy been discussed in a previous paper,[1] the solution to Eqs. (35) depends on the structure of the kernels which appear in the homogeneous terms (cf. Eq. (40)]. Although algebraically more involved, it can be shown by the same procedure as followed in the paper by GCGC that each of these terms has the following properties:

(a) The right eigenfunctions are 1, $\mathbf{p}_\gamma$, and p_γ^2 ($\gamma = 1, 2$) for each of the species.

(b) The left eigenfunctions with zero eigenvalue corresponding to the equation for species γ are 1, $\mathbf{p}_\gamma$, anh $\epsilon'_{\{2-\gamma\}\{\gamma-1\}}(\mathbf{q}_{\gamma 1}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$, the functional derivative of the total energy taken with respect to the two one-particle distribution functions.

(c) These left eigenfunctions are orthogonal to the inhomogeneous parts of Eqs. (35).

Under these conditions, the solutions to these equations exists, but are undetermined up to a linear combination of the solutions to the corresponding homogeneous equations. However, the five arbitrary constants in the solution can be uniquely given with the aid of the five subsidiary conditions (41)–(43). This standard procedure leads to the result that

$$\phi_{\{2-\gamma\}\{\gamma-1\}}(\mathbf{p}) = \sum_{\alpha=1}^{2} \mathscr{A}_{\gamma\alpha}\mathbf{p}_\gamma \cdot \operatorname{grad} \ln n_\alpha + \mathscr{C}_\gamma \mathbf{p}_\gamma \cdot \operatorname{grad} \ln \theta + \mathscr{D}_\gamma \operatorname{div} \mathbf{u} + \mathscr{F}_\gamma(\mathbf{p}_\gamma \mathbf{p}_\gamma)_s : (\operatorname{grad} \mathbf{u})_s, \qquad \gamma = 1, 2 \tag{44}$$

where the coefficients $\mathscr{A}_{\gamma\alpha}$, $\mathscr{C}_\gamma$, $\mathscr{D}_\gamma$, and $\mathscr{F}_\gamma$ ($\gamma, \alpha = 1, 2$) satisfy the following linear inhomogeneous integral equations:

$$\begin{pmatrix} \mathbf{A}_{\gamma\alpha} \\ \mathbf{C}_\gamma \end{pmatrix} = \sum_{\nu=1}^{2} \int d\mathbf{p}' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')\, \mathbf{p}_\nu' \begin{pmatrix} \mathscr{A}_{\gamma\alpha} \\ \mathscr{C}_\gamma \end{pmatrix}(p_\nu'), \qquad \gamma, \alpha = 1, 2 \tag{45}$$

$$D_\gamma = \sum_{\nu=1}^{2} \int d\mathbf{p}' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')\, \mathscr{D}_\gamma(p_\gamma'), \qquad \gamma = 1, 2 \tag{46}$$

$$\mathsf{F}_\gamma = \sum_{\nu=1}^{2} \int d\mathbf{p}' \, M_{\gamma,\{2-\nu\}\{\nu-1\}}(x, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')(\mathbf{p}_\gamma'\mathbf{p}_\gamma')_s\, \mathscr{F}_\gamma(p_\gamma'), \qquad \gamma = 1, 2 \tag{47}$$

subject to the new subsidiary conditions

$$\int d\mathbf{p}\, \mathbf{p} \sum_{\gamma=1}^{2} f^{e}_{\{2-\gamma\}\{\gamma-1\}} \boldsymbol{\phi}_\gamma \begin{pmatrix} \mathscr{A}_{\gamma\alpha} \\ \mathscr{C}_\gamma \end{pmatrix} = 0, \qquad \alpha = 1, 2 \tag{48}$$

$$\int d\mathbf{p}\, f^{e}_{\{2-\gamma\}\{\gamma-1\}} \mathscr{D}_\gamma = 0, \qquad \gamma = 1, 2 \tag{49}$$

In the derivation of Eqs. (44), use has been made of the fact that our system is considered to be an isotropic one, so that the tensorial character of the solution has been already accounted for in the usual way.[1]

In contrast to what happens in the one-component system, Eqs. (44) are not suitable for computing transport coefficients. Indeed, the thermodynamic forces represented by the gradients of the macroscopic variables which appear in them are not the conjugate ones to the fluxes defined by Eqs. (8) and (12) and

$$\mathbf{J}_7 = \mathbf{j} - \sum_{\alpha=1}^{2} (h_\alpha/m_\alpha)\, \mathbf{J}_\alpha\,, \tag{50}$$

where $\mathbf{j}$ is given by Eq. (18).

We know from irreversible thermodynamics[4] that such forces are $\boldsymbol{\Lambda}_1$, $\boldsymbol{\Lambda}_2$, and grad θ, where

$$\boldsymbol{\Lambda}_\gamma = (1/m_\gamma)[\operatorname{grad} \mu_\gamma + (s_\gamma/k) \operatorname{grad} \theta], \qquad \gamma = 1, 2 \tag{51}$$

μ_γ being the chemical potential per particle of species γ, s_γ the partial entropy per particle of species γ, and k Boltzmann's constant. Furthermore, h_α is the partial enthalpy per particle of species α. In the following, we shall discuss the structure of the solution to our set of integral equations in terms of the latter representation for the forces.

5. TRANSFORMATION OF THE CHAPMAN-ENSKOG SOLUTION

As was pointed out in the previous section, the computation of transport coefficients for a binary mixture of dense gases adequate for directly comparison with experiment, requires that the solution to the kinetic equations should be expressed in terms of $\boldsymbol{\Lambda}_\gamma$, grad θ, and grad $\mathbf{u}$. This implies that Eqs. (44) must be written as

$$\begin{aligned} \phi_{\{2-\gamma\}\{\gamma-1\}}(\mathbf{p}) = {} & \sum_{\alpha=1}^{2} \bar{\mathscr{A}}_{\gamma\alpha} \boldsymbol{\phi}_\gamma \cdot \boldsymbol{\Lambda}_\alpha + \bar{\mathscr{C}}_\gamma \boldsymbol{\phi}_\gamma \cdot \operatorname{grad} \ln \theta \\ & + \mathscr{D} \operatorname{div} \mathbf{u} + \mathscr{F}_\gamma(\boldsymbol{\phi}_\gamma \boldsymbol{\phi}_\gamma)_s : (\operatorname{grad} \mathbf{u})_s\,, \qquad \gamma = 1, 2 \end{aligned} \tag{52}$$

where the coefficients $\bar{\mathscr{A}}_{\gamma\alpha}$ and $\bar{\mathscr{C}}_{\gamma}$ are to be determined. For this purpose, we must eliminate the terms containing grad n_γ in our former solutions in favor of the Λ_γ. Since

$$\mu_\gamma = \mu_\gamma(n_a, n_b, \theta), \qquad \gamma = 1, 2 \tag{53}$$

the transformation is given by

$$\begin{bmatrix} \text{grad} \ln \theta \\ (1/\theta)\, m_a \mathbf{\Lambda}_a \\ (1/\theta)\, m_b \mathbf{\Lambda}_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ E_{10} & E_{11} & E_{12} \\ E_{20} & E_{21} & E_{22} \end{bmatrix} \cdot \begin{bmatrix} \text{grad} \ln \theta \\ \text{grad} \ln n_a \\ \text{grad} \ln n_b \end{bmatrix} \tag{54}$$

where the elements of the transformation matrix are given by

$$E_{\gamma 0} = \frac{2 s_\gamma}{k}, \qquad E_{\gamma\alpha} = \frac{n_\alpha}{\theta} \frac{\partial \mu_\gamma}{\partial n_\alpha}, \qquad \alpha, \gamma = 1, 2 \tag{55}$$

Since we are interested in the inverse of the transformation, we must require that the transformation matrix has an inverse, i.e., the determinant of such a matrix must be different from zero. By irreversible thermodynamics, we know that both sets of thermodynamic forces are linearly independent and therefore using a well-known theorem[(4)] the inverse of the transformation matrix must exist.[4] Hence,

$$\begin{bmatrix} \text{grad} \ln \theta \\ \text{grad} \ln n_a \\ \text{grad} \ln n_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} \text{grad} \ln \theta \\ (m_a/\theta)\, \mathbf{\Lambda}_a \\ (m_a/\theta)\, \mathbf{\Lambda}_b \end{bmatrix} \tag{56}$$

where the elements of the inverse matrix are given by

$$\begin{aligned} b_{\alpha 0} &= \frac{1}{\Delta} \frac{2}{k\theta}\, n_{3-\alpha} \left(s_{3-\alpha} \frac{\partial \mu_\alpha}{\partial n_{3-\alpha}} - s_\alpha \frac{\partial \mu_{3-\alpha}}{\partial n_{3-\alpha}} \right) \\ b_{\alpha\gamma} &= \frac{(-)^{\alpha+\gamma}}{\Delta} \frac{n_{3-\alpha}}{\theta} \frac{\partial \mu_{3-\gamma}}{\partial n_{3-\alpha}}, \qquad \alpha, \gamma = 1, 2 \end{aligned} \tag{57}$$

[4] Direct calculation of the determinant yields as a result that this condition is equivalent to the requirement that

$$\Delta \equiv \frac{\partial \mu_a}{\partial n_a} \frac{\partial \mu_b}{\partial n_b} - \frac{\partial \mu_a}{\partial n_b} \frac{\partial \mu_b}{\partial n_a} \neq 0$$

This has already been proved explicitly to the first order in the density in Ref. 5.

Making use of these results, one finds that the new coefficients appearing in Eqs. (52) are given by

$$\bar{\mathscr{A}}_{\gamma\alpha} = \sum_{\nu=1}^{2} \mathscr{A}_{\gamma\alpha} b_{\nu\alpha}\,, \qquad \alpha, \gamma = 1, 2 \tag{58}$$

$$\bar{\mathscr{C}}_{\gamma} = \mathscr{C}_{\gamma} + \sum_{\alpha=1}^{2} \mathscr{A}_{\gamma\alpha} b_{\alpha 0}\,, \qquad \gamma = 1, 2 \tag{59}$$

These results may be used to compute transport coefficients and this will be the subject of the next section.

6. THE TRANSPORT COEFFICIENTS

In order to calculate transport coefficients, it is necessary to obtain explicit expressions for the diffusion currents, the heat flux, and the stress tensor obtained in Section 2 which are localized around the observation point $\mathbf{q}_{\gamma 1}$, and which are also linearized to first order in the gradients of the system. Such expressions can be derived following the procedure which was discussed in Ref. 1. Using these results together with the formulas for the fluxes given by Eqs. (8), (12), and (50) and the solutions to the inhomogeneous integral equations obtained in the previous section, i.e., eq. (52), we obtain the following results.

(a) Diffusion current:

$$\mathbf{J}_{\alpha} = -L_{\alpha 0} \operatorname{grad} \ln \theta - \sum_{\gamma=1}^{2} L_{\alpha\gamma} \mathbf{\Lambda}_{\gamma}\,, \qquad \alpha = 1, 2 \tag{60}$$

where

$$L_{\alpha 0} = -\frac{1}{3m_{\alpha}} \int d\mathbf{p}_{\alpha}\, \mathbf{p}_{\alpha}{}^{2} f^{e}_{\{2-\alpha\}\{\alpha-1\}}(p_{\alpha})\, \bar{\mathscr{C}}_{\alpha}(p_{\alpha}), \qquad \alpha = 1, 2 \tag{61}$$

$$L_{\alpha\gamma} = -\frac{1}{3m_{\alpha}} \int d\mathbf{p}\, p_{\alpha}{}^{2} f^{e}_{\{2-\alpha\}\{\alpha-1\}}(p_{\alpha})\, \bar{\mathscr{A}}_{\alpha\gamma}(p_{\alpha}), \qquad \alpha, \gamma = 1, 2 \tag{62}$$

The coefficients $L_{\alpha 0}$ are the thermodiffusion or Soret coefficients, and $L_{\alpha\gamma}$ are the mutual diffusion coefficients.

(b) Heat flux:

$$\mathbf{J}_{q} = -L_{00} \operatorname{grad} \ln \theta - \sum_{\mu=1}^{2} L_{0\mu} \mathbf{\Lambda}_{\mu} \tag{63}$$

where

$$
\begin{aligned}
L_{00} = & -\sum_{\gamma=1}^{2} \frac{1}{6m_\gamma{}^2} \int d\mathbf{p}_{\gamma 1}\, p_{\gamma 1}^4 f^e_{\{2-\gamma\}\{\gamma-1\}}(p_{\gamma 1})\, \mathscr{C}_\gamma(p_{\gamma 1}) \\
& + \sum_{\gamma=1}^{2} \frac{h_\gamma}{m_\gamma} L_{\gamma 0} - \frac{1}{12} \sum_{\alpha=1}^{2} \sum_{\gamma=1}^{2} \sum_{\nu=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \int d\mathbf{p}' \\
& \times \frac{\varphi'_{\gamma\alpha}}{\gamma_{\gamma\alpha}} \mathbf{r}_{\gamma\alpha} \cdot \left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma} + \frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha} \right) f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')\, \mathbf{r}_{\gamma\alpha} \cdot \mathbf{p}_\nu{}' \mathscr{C}_\nu(p') \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)} \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \\
& - \frac{1}{12} \sum_{\alpha=1}^{2} \sum_{\gamma=1}^{2} \sum_{\nu=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \int dx' \frac{\varphi'_{\gamma\alpha}}{\gamma_{\gamma\alpha}} \mathbf{r}_{\gamma\alpha} \cdot \left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma} + \frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha} \right) \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma-\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \\
& \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')\, \omega_\gamma(p')\, \mathbf{r}_{\gamma\alpha} \cdot (\mathbf{q}' - \mathbf{q}_{\gamma 1}) \\
& - \frac{1}{6} \sum_{\alpha=1}^{2} \sum_{\gamma=1}^{2} \sum_{\nu=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \int d\mathbf{p}'\, \varphi_{\gamma\alpha} \frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \cdot \mathbf{p}_\nu{}' \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \\
& \times f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')\, \mathscr{C}_\nu(\mathbf{p}') \\
& - \frac{1}{6} \sum_{\alpha=1}^{2} \sum_{\gamma=1}^{2} \sum_{\nu=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \int dx'\, \varphi_{\gamma\alpha} \frac{\mathbf{p}_{\gamma 1}}{m_\gamma} \cdot (\mathbf{q}' - \mathbf{q}_{\gamma 1}) \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}}) \\
& \times \omega_\gamma(p') f^e_{\{2-\nu\}\{\nu-1\}}(\mathbf{p}')
\end{aligned}
\tag{64}
$$

and

$$
\begin{aligned}
L_{0\mu} = & -\sum_{\gamma=1}^{2} \frac{1}{6m_\gamma{}^2} \int d\mathbf{p}_{\gamma 1}\, p_{\gamma 1}^4 f^e_{\{2-\gamma\}\{\gamma-1\}}(p_{\gamma 1})\, \mathscr{A}_{\gamma\mu}(p_{\gamma 1}) \\
& + \sum_{\gamma=1}^{2} \frac{h_\gamma}{m_\gamma} L_{\gamma\mu} - \frac{1}{12} \sum_{\alpha=1}^{2} \sum_{\gamma=1}^{2} \sum_{\nu=1}^{2} \int d\mathbf{p}_{\gamma 1} \int dx_{\alpha,\xi(\alpha,\gamma)} \int d\mathbf{p}' \\
& \times \frac{\varphi'_{\gamma\alpha}}{r_{\gamma\alpha}} \mathbf{r}_{\gamma\alpha} \cdot \left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma} + \frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha} \right) f^e_{\{2-\nu\}\{\nu-1\}}(p')\, \mathbf{r}_{\gamma\alpha} \cdot \mathbf{p}_\nu{}' \mathscr{A}_{\nu\mu}(p') \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1} + \mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})
\end{aligned}
$$

$$-\frac{1}{12}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma 1}\int dx_{\alpha,\xi(\alpha,\nu)}\int dx'\,\frac{\varphi'_{\gamma\alpha}}{\gamma_\alpha}\,\mathbf{r}_{\gamma\alpha}\cdot\left(\frac{\mathbf{p}_{\gamma 1}}{m_\gamma}+\frac{\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}}{m_\alpha}\right)$$

$$\times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$

$$\times f^e_{\{2-\nu\}\{\nu-1\}}(p')\,\sigma_{\gamma\mu}(p')\,\mathbf{r}_{\gamma\alpha}\cdot(\mathbf{q}'-\mathbf{q}_{\gamma 1})$$

$$-\frac{1}{6}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma 1}\int dx_{\alpha,\xi(\alpha,\gamma)}\int d\mathbf{p}'\,\varphi_{\gamma\alpha}\,\frac{\mathbf{p}_{\gamma 1}}{m_\gamma}\cdot\mathbf{p}_\nu'$$

$$\times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$

$$\times f^e_{\{2-\nu\}\{\nu-1\}}(p')\,\bar{\mathscr{A}}_{\nu\mu}(p')$$

$$-\frac{1}{6}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma 1}\int dx_{\alpha,\xi(\alpha,\gamma)}\int dx'\,\varphi_{\gamma\alpha}\,\frac{\mathbf{p}_{\gamma 1}}{m_\gamma}\cdot(\mathbf{q}'-\mathbf{q}_{\gamma 1})$$

$$\times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$

$$\times\ \sigma_{\gamma\mu}(p')\,f^e_{\{2-\nu\}\{\nu-1\}}(p') \tag{65}$$

In these expressions, we have written

$$\omega_\gamma(p) = (p_\gamma^2/2m_\gamma\theta) - \tfrac{3}{2} - b_{\gamma 0}, \tag{66}$$

and

$$\sigma_{\gamma\mu} = (b_{\gamma\mu}/\theta)\,m_\mu \tag{67}$$

The coefficient L_{00} is the heat conductivity and the coefficients $L_{0\mu}$ are the Dufour coefficients.

(c) Stress tensor:

$$\mathsf{P} = p\mathsf{I} - 2\eta(\text{grad }\mathbf{u})_s - \zeta\,\text{div }\mathbf{u}\,\mathsf{I} \tag{68}$$

where

$$\eta = -\frac{1}{15}\sum_{\gamma=1}^{2}\int d\mathbf{p}_{\gamma 1}\,\frac{p_{\gamma 1}^4}{m_\gamma}\,f^e_{\{2-\gamma\}\{\gamma-1\}}(p_{\gamma 1})\,\mathscr{F}_\gamma(p_{\gamma 1})$$

$$+\frac{1}{20}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma 1}\int d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}\int d\mathbf{r}_{\gamma\alpha}\int d\mathbf{p}'$$

$$\times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma 1}, \mathbf{q}_{\gamma 1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma 1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, \mathbf{p}' \mid f^e_{\{1\}\{0\}}, f^e_{\{0\}\{1\}})$$

$$\times\ \frac{\varphi'_{\gamma\alpha}}{\gamma_{\gamma\alpha}}\,f^e_{\{2-\nu\}\{\nu-1\}}(p_\gamma')\,\mathscr{F}_\gamma(p_\gamma')\left[(\mathbf{r}_{\gamma\alpha}\cdot\mathbf{p}_\gamma')^2-\frac{1}{3}\,\gamma_{\gamma\alpha}^2 p_\gamma'^2\right]$$

$$
\begin{aligned}
& -\frac{1}{20\Theta}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma1}\int d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}\int d\mathbf{r}_{\gamma\alpha}\int dx' \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma1}, \mathbf{q}_{\gamma1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^{e}_{\{1\}\{0\}}, f^{e}_{\{0\}\{1\}}) \\
& \times \frac{\varphi'_{\gamma\alpha}}{r_{\gamma\alpha}} f^{e}_{\{2-\nu\}\{\nu-1\}}(p_\gamma')[\mathbf{r}_{\gamma\alpha}\cdot(\mathbf{q}'-\mathbf{q}_{\gamma1})\,\mathbf{r}_{\gamma\alpha}\cdot p_\gamma' \\
& -\tfrac{1}{3}\gamma^2_{\gamma\alpha} p_\gamma'\cdot(\mathbf{q}'-\mathbf{q}_{\gamma1})]
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
\zeta = & -\frac{1}{3}\sum_{\gamma=1}^{2}\int d\mathbf{p}_{\gamma1}\frac{p_{\gamma1}^2}{m_\gamma} f^{e}_{\{2-\gamma\}\{\gamma-1\}}(p_{\gamma1})\,\mathscr{D}_\gamma(p_{\gamma1}) \\
& +\frac{1}{12}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma1}\int d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}\int d\mathbf{r}_{\gamma\alpha}\int d\mathbf{p}' \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma1}, \mathbf{q}_{\gamma1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, \mathbf{p}' \mid f^{e}_{\{1\}\{0\}}, f^{e}_{\{0\}\{1\}}) \\
& \times \varphi'_{\gamma\alpha} r_{\gamma\alpha} f^{e}_{\{2-\nu\}\{\nu-1\}}(p_\gamma')\,\mathscr{D}_\gamma(p_\gamma') \\
& +\frac{1}{36\Theta}\sum_{\alpha=1}^{2}\sum_{\gamma=1}^{2}\sum_{\nu=1}^{2}\int d\mathbf{p}_{\gamma1}\int d\mathbf{p}_{\alpha,\xi(\alpha,\gamma)}\int d\mathbf{r}_{\gamma\alpha}\int dx' \\
& \times f'_{\{4-\gamma-\alpha\}\{\gamma+\alpha-2\},\{2-\nu\}\{\nu-1\}}(\mathbf{q}_{\gamma1}, \mathbf{q}_{\gamma1}+\mathbf{r}_{\gamma\alpha}, \mathbf{p}_{\gamma1}, \mathbf{p}_{\alpha,\xi(\alpha,\gamma)}, x' \mid f^{e}_{\{1\}\{0\}}, f^{e}_{\{0\}\{1\}}) \\
& \times \varphi'_{\gamma\alpha} r_{\gamma\alpha} f^{e}_{\{2-\nu\}\{\nu-1\}}(p_\gamma')\,p_\gamma'\cdot(\mathbf{q}'-\mathbf{q}_{\gamma1})
\end{aligned}
\tag{70}
$$

The coefficients η and ζ are the shear and bulk viscosities, respectively.

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